Double Integrals and Change of Order of Integration

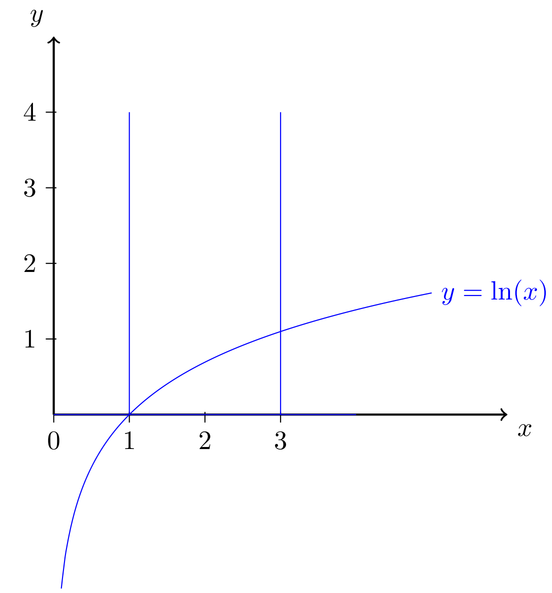
Calculate the volumes of solids by applying double integrals:

$$\int\int_D e^{2y} dx dy$$

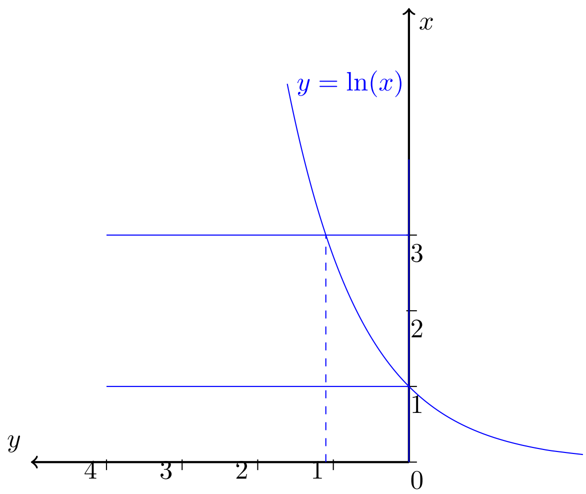
With $D = \{(x, y)/1 \leq x \leq 3 \wedge 0 \leq y \leq \ln(x)\}$

Solution

First, we plot:



Rotated graph:



I find the intersection point by evaluating the function at 3: $\ln(3) = 1.099$. I solve for $y = \ln(x)$. $e^y = x$
We set up the double integral

$$\int_0^{\ln(3)} \int_{e^y}^{3} e^{2y} dx dy$$

I solve the first integral:

$$xe^{2y}$$

Evaluate at the bounds:

$$3e^{2y} - e^y e^{2y} = 3e^{2y} - e^{3y}$$

Integrate with respect to y

$$3e^{2y}\frac{1}{2} - e^{3y}\frac{1}{3}$$

Evaluate at the bounds:

$$3e^{2\ln(3)}\frac{1}{2} - e^{3\ln(3)}\frac{1}{3} - [3/2 - 1/3] = 10/3$$

Now we change the order of integration

$$\int_1^3 \int_0^{\ln(x)} e^{2y} dy dx$$

Perform the first integral:

$$e^{2y}\frac{1}{2}$$

Evaluate at the bounds:

$$e^{2\ln(x)}\frac{1}{2} - 1/2 = e^{\ln(x^2)}\frac{1}{2} - 1/2 = \frac{x^2}{2} - 1/2$$

Perform the second integral:

$$\frac{x^3}{6} - x/2$$

Evaluate at the bounds:

$$27/6 - 3/2 - [1/6 - 1/2] = 10/3$$